Calculus Warm Up #2-3

Find the sum:

1) $\sum_{i=1}^{10} i(i^2+1)$

2) $\sum_{i=1}^{15} (2i-3)$

HW Questions: p. 248

In Exercises 1–8, find the given sum.

1. $\sum_{i=1}^{5} (2i + 1)$

3. $\sum_{k=0}^{4} \frac{1}{k^2 + 1}$

5. $\sum_{k=1}^{4} c$

7. $\sum_{i=1}^{4} [(i - 1)^2 + (i + 1)^3]$

In Exercises 9–18, use sigma notation to write the given sum.

9. $\frac{1}{3(1)} + \frac{1}{3(2)} + \frac{1}{3(3)} + \cdots + \frac{1}{3(9)}$

11. $\left[2\left(\frac{1}{8}\right) + 3\right] + \left[2\left(\frac{2}{8}\right) + 3\right] + \cdots + \left[2\left(\frac{8}{8}\right) + 3\right]$

13. $\left[\left(\frac{1}{6}\right)^2 + 2\right]\left(\frac{1}{6}\right) + \cdots + \left[\left(\frac{6}{6}\right)^2 + 2\right]\left(\frac{1}{6}\right)$

15. $\left[\left(\frac{2}{n}\right)^3 - \frac{2}{n}\right]\left(\frac{2}{n}\right) + \cdots + \left[\left(\frac{2n}{n}\right)^3 - \frac{2n}{n}\right]\left(\frac{2}{n}\right)$

17. $\left[2\left(1 + \frac{3}{n}\right)^2\right]\left(\frac{3}{n}\right) + \cdots + \left[2\left(1 + \frac{3n}{n}\right)^2\right]\left(\frac{3}{n}\right)$

In Exercises 19–24, use the properties of sigma notation and summation formulas to evaluate the given sum.

19. $\sum_{i=1}^{20} 2i$

21. $\sum_{i=1}^{20} (i-1)^2 = \sum_{i=1}^{19} i^2$

25. Find the limit of S_n as $n \to \infty$

$$S_n = \frac{4}{3n^3}\left(2n^3 + 3n^2 + n\right)$$

From yesterday:

<u>Properties</u>

$$\sum_{i=1}^{n} ka_i = k\sum_{i=1}^{n} a_i \text{ , where } k = \text{constant}$$

$$\sum_{i=1}^{n} (a_i \pm b_i) = \sum_{i=1}^{n} a_i \pm \sum_{i=1}^{n} b_i$$

From yesterday: <u>Summation Formulas</u>

1. $\sum_{i=1}^{n} c = cn$

2. $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

3. $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

4. $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

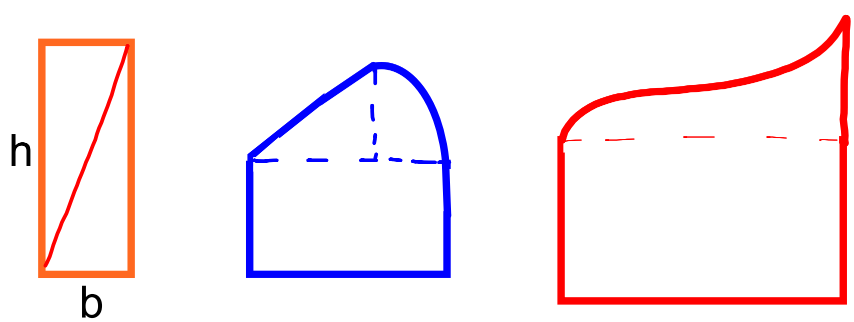
* Notice this is rule #2 squared.

5.2 Day 2

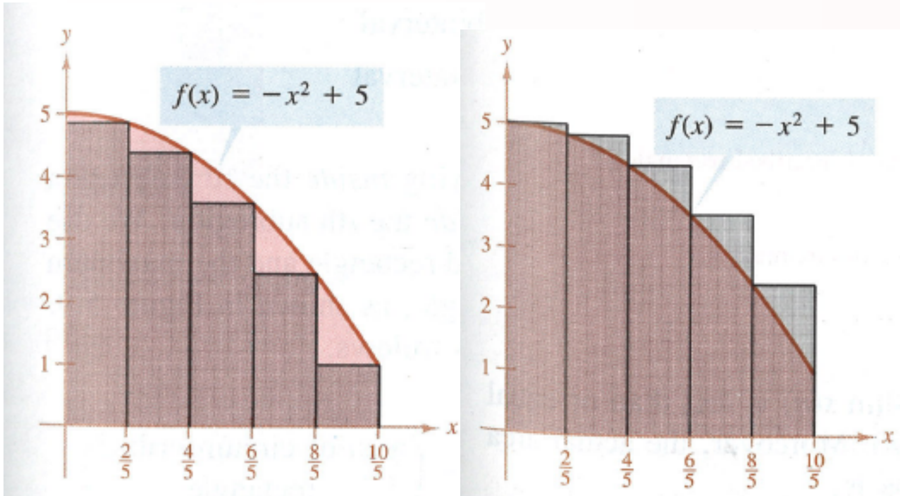
-Area

-The Area of a Plane Region

-Upper and Lower Sums



Approximating Area under a curve using rectangles

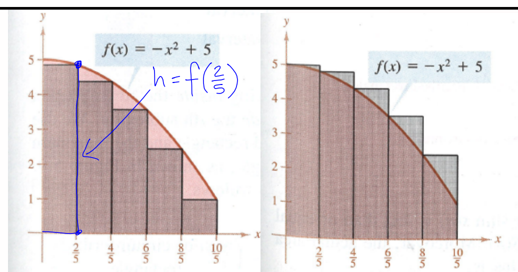


Area is an Underestimation

Area is an Overestimation

lower sum

upper sum



Underestimation: using Right endpoints

$$A = \frac{2}{5}\left[f\left(\frac{2}{5}\right)\right] + \frac{2}{5}\left[f\left(\frac{4}{5}\right)\right] + \frac{2}{5}\left[f\left(\frac{6}{5}\right)\right] + \frac{2}{5}\left[f\left(\frac{8}{5}\right)\right] + \frac{2}{5}\left[f\left(\frac{10}{5}\right)\right]$$

$$A = 6.48$$

Overestimation: using Left endpoints

$$A = \frac{2}{5}\left[f(0) + f\left(\frac{2}{5}\right) + f\left(\frac{4}{5}\right) + f\left(\frac{6}{5}\right) + f\left(\frac{8}{5}\right)\right]$$

$$= 8.08$$

$$6.48 < \text{Actual Area} < 8.08$$

lower sum

upper sum

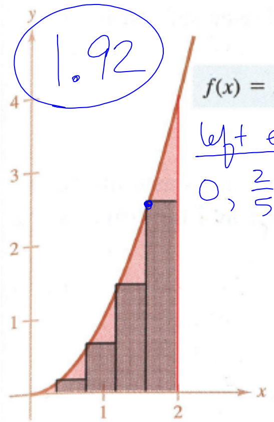
$$s_n < A < S_n$$

Using 5 rectangles of uniform width, find the upper and lower sums for the region bounded by f(x) and the x-axis on [0, 2].

let Δx = width of rectangle

$\Delta x = \frac{2-0}{5}$

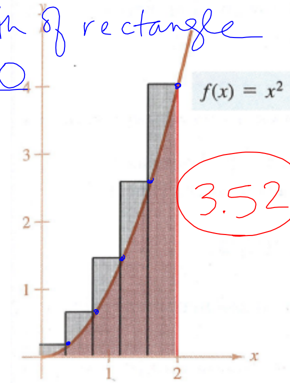
1.92



Inscribed rectangles

left endpts.

$0, \frac{2}{5}, \frac{4}{5}, \frac{6}{5}, \frac{8}{5}$



3.52

Circumscribed rectangles

Rt endpts

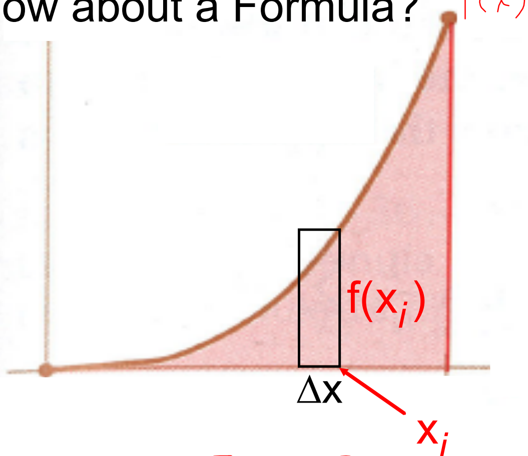
$\frac{2}{5}, \frac{4}{5}, \frac{6}{5}, \frac{8}{5}, \frac{10}{5}$

$A = \frac{2}{5}\left[f(0) + f\left(\frac{2}{5}\right)\ldots\right.$

$A = \frac{2}{5}\left[f\left(\frac{2}{5}\right) + f\left(\frac{4}{5}\right) + \ldots + f\left(\frac{10}{5}\right)\right]$

$$\boxed{1.92 < A < 3.52}$$

How about a Formula?



Let Δx = width of a rectangle

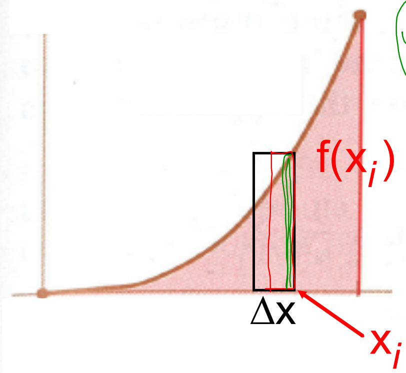
x_i = right x-value of the rectangle

$f(x_i)$= height of the rectangle

$A_{rect.} = \Delta x\left[f(x_i)\right]$

for n rectangles: $A = \sum_{i=1}^{n} \Delta x\left[f(x_i)\right]$

What would make it more accurate?



$\lim\limits_{\Delta x \to 0}$

Let Δx = width of a rectangle

x_i = right x-value of the rectangle

$f(x_i)$ = height of the rectangle

for n rectangles: $A = \sum_{i=1}^{n} \Delta x \left[f(x_i) \right]$

$$\lim_{n \to \infty} \sum_{i=1}^{n} \Delta x \left[f(x_i) \right]$$

DEFINITION OF AREA OF A REGION IN THE PLANE

Let f be continuous and nonnegative on the interval $[a, b]$. The **area** of the region bounded by the graph of f, the x-axis, and the vertical lines $x = a$ and $x = b$ is

$$\text{area} = \lim_{n \to \infty} \sum_{i=1}^{n} f(c_i)\Delta x, \quad x_{i-1} \le c_i \le x_i$$

where $\Delta x = (b - a)/n$.

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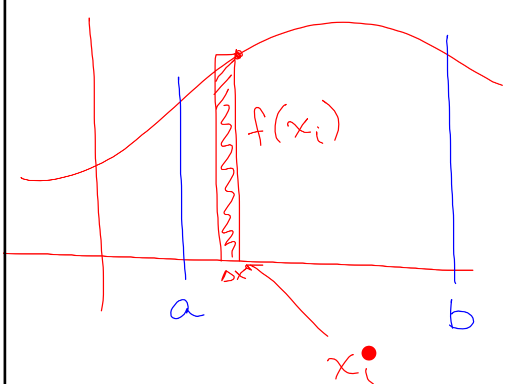
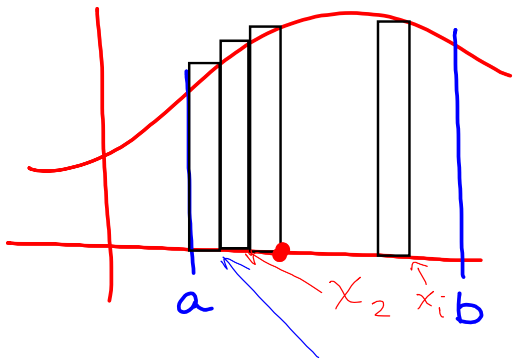


$$A = \lim_{n \to \infty} \sum_{i=1}^{n} \Delta x \left[f(x_i) \right]$$

$$\boxed{\Delta x = \frac{b-a}{n}}$$

Naming x_i



$$x_i = a + \Delta x \cdot i$$

$$x_1 = a + \Delta x$$

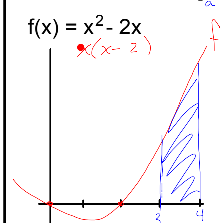
$$x_2 = a + \Delta x \cdot 2$$

$$x_3 = a + \Delta x \cdot 3$$

Find the area of the region bounded by f(x) and the x-axis on [3, 4]

$f(x) = x^2 - 2x$

$x(x-2)$



$$\Delta x = \frac{b-a}{n}$$

$$\Delta x = \frac{4-3}{n} = \frac{1}{n}$$

$$x_i = a + \Delta x \cdot i$$

$$= 3 + \frac{1}{n} \cdot i$$

$$x_i = 3 + \frac{i}{n}$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\left[f\left(3+\frac{i}{n}\right)\right]$$

$$= \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \left[\left(3+\frac{i}{n}\right)^2 - 2\left(3+\frac{i}{n}\right)\right]$$

$$= \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \left(9 + \frac{6i}{n} + \frac{i^2}{n^2} - 6 - \frac{2i}{n}\right)$$

$$= \lim_{n\to\infty} \frac{1}{n}\left(\sum 3 + \frac{4}{n}\sum i + \frac{1}{n^2}\sum i^2\right)$$

$$\lim_{n\to\infty} \frac{1}{\cancel{n}}\left(3\cancel{n} + \frac{\cancel{4}^{2}}{n} \cdot \frac{\cancel{n}(n+1)}{\cancel{2}} + \frac{1}{n^2} \cdot \frac{\cancel{n}(n+1)(2n+1)}{6}\right)$$

$$\lim_{n\to\infty}\left(3 + \frac{2n+2}{n} + \frac{2n^2+3n+1}{6n^2}\right)$$

$$\lim_{n\to\infty}\left(3 + 2 + \cancel{\frac{2}{n}} + \frac{1}{3} + \cancel{\frac{1}{2n}} + \cancel{\frac{1}{6n^2}}\right)$$

$$A = \frac{16}{3}$$

Summary:

when given n = a particular number of subintervals

find $\Delta x = \frac{b-a}{n}$ on $[a, b]$

find Rt endpts $\Rightarrow$ $x_i = a + \Delta x \cdot i$

left endpts $\Rightarrow$ $x_i = a + \Delta x(i-1)$

approx area

$$A = \Delta x\left[f(x_1) + f(x_2) + \dots + f(x_n)\right]$$

Same as:

$$A = \sum_{i=1}^{n} \Delta x\left[f(x_i)\right]$$

Finding Actual Area

Let $\Delta x \to 0$ approach   Same as let $n \to \infty$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} \Delta x\left[f(x_i)\right]$$

* Limit Definition of the Area under a Curve.

HW:

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